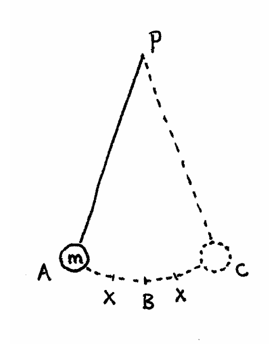
THE PHYSICS OF THE TWO-STAGE PENDULUM OSCILLATOR

(1) A simple pendulum

(a) As a pendulum moves from *A* to *B* the potential energy which it possesses at *A* decreases and is converted into kinetic energy. From *B* to *C* the kinetic energy is converted back into potential energy. If no energy is lost the PE at *A* is equal to the PE at *C* and, at all points on the path *ABC*, the total energy, PE + KE, is constant.



The velocity at B is given by $v = \sqrt{2gh}$ where *h* is the vertical height between *A* and *B*. This velocity depends on the value of *g* as well as on the value of *h*. The period of the pendulum is $T = 2\pi\sqrt{l/g}$ and also depends on *g*.

If energy is lost or removed from the system the amplitude decreases and energy needs to be added to the system to maintain a constant amplitude.

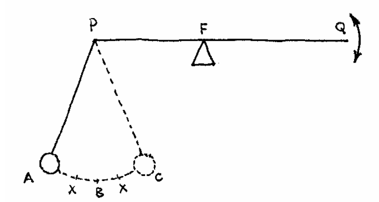
For the simple pendulum the downwards component of the tension in the supporting string is less than *mg* from *A* to *X* and from *X* to *C* and larger than *mg* between *X* and *X* as the bob moves through *B*.

(b) If the point, *P*, is allowed to accelerate vertically, the motion of the pendulum changes. If it moves upwards with an acceleration, *a*, the effective value of g is $g' = g + a$ and the velocity at *B* is larger than the velocity if *P* is stationary. The period of the pendulum decreases.

If *P* accelerates downwards with an acceleration, *a*, the effective value of *g* is $g' = g - a$ and the velocity of the bob at *B* is smaller than if *P* were stationary and the period of the pendulum increases.

If the pendulum is allowed to fall freely under the action of gravity (so that $a = g$) the pendulum would stop swinging so the velocity at B would be zero.

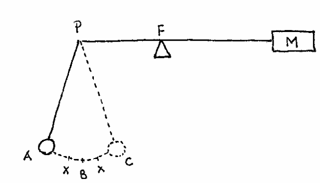
(c) This effect can be demonstrated as follows. Attach the pendulum to the end of a light rod, *PQ*, and place the centre of the rod on a fulcrum, *F,* while supporting *Q* with your hand. Set the pendulum swinging.



As the bob moves through the lowest point, *B*, move *Q* downwards and when the bob reaches *C* move *Q* upwards. Repeat this on the reverse journey. Over a number of cycles the end result is that, because *P* accelerates upwards when the bob is at *B* the bob speeds up. At *C* (now higher than before) the bob is at rest so it cannot slow down any further when *P* accelerates downwards. The end result is that the amplitude of the pendulum (as well as its total energy) increases because of the work done by the external force which moves *Q* up and down. If the point *P* is moved down when the pendulum bob is at B the pendulum reduces its amplitude (and its total energy).

(2) A pendulum connected to a free, balanced lever

If the pendulum is at rest the lever is at rest in a horizontal position because of the mass, *M*, on the other side. My expectation about the behaviour of the system when the pendulum is oscillating is as follows.



The lever system is balanced when the vertically downward force at *P* is equal to the weight of the pendulum bob, *mg*. When the bob is moving between *A* and *X* the vertically downwards component of the tension in the supporting string is less than *mg*. Thus the point *P* begins to accelerate upwards so the velocity of the bob becomes greater than for a pendulum where *P* is fixed.

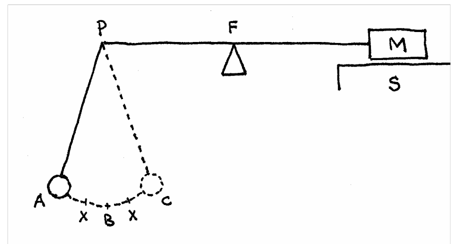
Between *X* and *X* this tension component is greater than *mg* so the point *P* begins to accelerate downwards causing *v* to decrease.

From *X* to *C* and *C* to *X*, *P* accelerates upwards again. Thus the mass , *M*, and the point *P* move up and down twice for one oscillation of the pendulum. In this motion the sum of the potential energies and the kinetic energies of both masses is constant.

(3) The two-stage oscillator

In the two-stage oscillator the mass, *M*, rests on a horizontal surface, *S*, so the lever is horizontal whenever the pendulum bob is between *A* and *X* or *C* and *X*. In this region the bob moves as it does in the simple pendulum with fixed *P* described in (1).

When the bob is between *X* and *X* the mass, *M*, is lifted off the surface, *S*, and the point, *P*, accelerates downwards. The velocity of the bob becomes less than the velocity for an equivalent simple pendulum.



When the bob moves from *X* to C the mass, *M*, accelerates downwards. When it reaches S it suddenly stops so the point, *P*, moving upwards also suddenly stops (that is, it has a large downward acceleration) so the velocity of the bob decreases.

In this system there is an external force which does work (or which has work done on it), namely, the upward force from S which reduces the total energy of the system. Thus, left to itself, the pendulum should slow down (independently of the effects of forces such as friction).

If, in addition, you take energy out of the system (for example, through your hand generators) the pendulum will slow down even more quickly.

I would expect that to keep the pendulum moving with the same amplitude, you need to put into the system the same amount of energy as that which has been lost or taken out.

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